Analele Universităţii Oradea
Fasc. Matematica, Tom XXVIII (2021), Issue No. 2, 77–94

ASYMPTOTIC ANALYSIS FOR A REACTION DIFFUSION EQUATION WITH A LOCALIZED NONLINEAR SOURCE TERM AND HOMOGENEOUS BOUNDARY CONDITIONS

N'GUESSAN KOFFI[1] , HALIMA NACHID[2], TOURÉ KIDJÉGBO AUGUSTIN[3]

Abstract. In this paper, we study a numerical approximation for the following localized nonlinear reaction-diffusion equation

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) + f(u(a_0,t)), \quad (x,t) \in (-1,1)\times(0,T), \\ u(-1,t) = 0, \quad u_x(1,t) = 0, \quad t \in (0,T), \\ u(x,0) = u_0(x) \geq 0, \quad x \in [-1,1], \end{cases}$$

where $f(s)$ is a positive, increasing, c^2 convex function for the nonnegative values of s, $\int_c^{+\infty} \frac{ds}{f(s)} < +\infty$ for $c > 0$, $a_0 = 1$, $u_0 \in C^2([-1,1])$, $u_0(-1) = 0$, $u_0'(1) = 0$.

We find some conditions under which the solution of a discrete form of the above problem blows up in a finite time and estimate its discrete blow-up time. We also prove the convergence of the numerical blow-up time to the theoretical one. Finally, we give some numerical results to illustrate our analysis.

References

[1] L. M. Abia, J. C. López-Marcos and J. Martinez, On the blow-up time convergence of semidiscretizations of reaction-diffusion equations, *Appl. Numer. Math.*, **26 (4)** (1998), 399-414.

[2] L. M. Abia, J. C. López-Marcos and J. Martínez, Blow-up for semidiscretizations of reaction-diffusion equations, *Appl. Numer. Math.*, **20 (1-2)** (1996), 145-156.

[3] G. Acosta, J. Fernandez Bonder, P. Groisman and J. D. Rossi, Simultaneous vs non-simultaneous blow-up in numerical approximations of a parabolic non-linear boundary conditions, *M2AN Math. Model. Numer. Anal.*, **36 (1)** (2002) 55-68.

[4] G. Acosta, J. Fernandez Bonder, P. Groisman and J. D. Rossi, Numerical approximation of a parabolic problem with a nonlinear boundary conditions in several space dimensions, *Disc. Cont. Dyn. Syst.*, **2(2)** (2002), 279-294.

[5] G. Acosta, R. Duran and J. D. Rossi, An adaptive time step procedure for a parabolic problem with blow-up, *Computing*, **68(4)** (2002), 343-373.

[6] C. Bandle and H. Brunner, Blow-up in diffusion equations: a survey. *J. Comput. Appl. Math.*, **97(1-2)** (1998), 3-22.

[7] M. Berger and R. V. Kohn, A rescaling algorithm for the numerical calculation of blowing up solutions, *Comm. Pure Appl. Math.*, **41(6)** (1998), 841-863.

[8] K. Bimpong-Beta, P. Ortoleva and J. Rossi, Far-form equilibrium phenomenon at local sites of reactions, *J. Chem. Physic*, **60 (8)** (1974), 3124-3133.

[9] T. K. Boni, Sur l'explosion et le comportement asymptotique de la solution d'une équation parabolique semi-linéaire du second ordre, *C.R.A.S*, Serie I, **326 (3)** (1998), 317-322.

[10] T. K. Boni, Extinction for discretizations of some semilinear parabolic equations, *C.R.A.S*, Serie I, **333** (2001), 75-80.

[11] T. K. Boni, On blow-up and asymptotic behavior of solutions to a nonlinear parabolic equation of second order with nonlinear boundary conditions, *Comment. Math. Univ. Comenian*, **40** (1999), 457-475.

2010 *Mathematics Subject Classification.* 35K57, 35B40, 35B44, 35B50, 65M06.

Key words and phrases. Reaction Diffusion Equation, localized nonlinear source term, discretizations, convergence, numerical blow-up time.

[12] C. Brandle, P. Groisman and J. D. Rossi, Fully adaptive methods, *Math. Models Methods Appl. Sci.* **14 (10)** (2004), 1425-1450.
[13] C. Brandle, F. Quiros and J. D. Rossi, An adaptive numerical method to handle blow-up in a parabolic system, *Numerische Mathematik*. **102 (1)** (2005), 39-59.
[14] H. Brezis, Analyse fonctionnelle, théorie et application, *Collection mathematiques appliquées pour la maitrise, Masson, Paris.* .
[15] H. Brezis, T. Cazenave, Y. Martel and A. Ramiandrisoa, Blow-up of $u_t = u_{xx} + g(u)$ revisited, *Adv. Diff. Eq.*, **1** (1996), 73-90.
[16] C. J. Budd, W. Huang and R. D. Russel, Moving mesh methods for problems with blow-up, *SIAM J. Sci Comput.*, **17(2)** (1996), 305-327.
[17] J. M. Chadam and H. M. Yin, A diffusion equation with localized chemical reactions, *Proc. Edinb. Math. Soc.*, **37(1)** (1994), 101-118.
[18] Y. G. Chen, Asymptotic behaviours of blowing up solutions for finite difference analogue of $u_t = u_{xx} + u^{1+\alpha}$, *J. Fac. Sci. Univ. Tokyo, Sec IA, Math.*, **33** (1986), 541-574.
[19] A. De. Pablo, M. LLanos and R. Ferreira, Numerical blow-up for p-Laplacian equation with a nonlinear source, *Proceedings of Equatdiff.*, **11** (2005), 363-367.
[20] R. G. Duran, J. I Etcheverry and J. D. Rossi, Numerical approximations of a parabolic problem with nonlinear boundary conditions, *Discrete Contin. Dynam. Systems*, **4 (3)** (1998), 497-506.
[21] C. M. Elliot and A. M. Stuart, Global dynamics of discrete semilinear parabolic equations , *SIAM J. Numer. Anal.*, **30(6)** (1993), 1622-1663.
[22] J. Fernandez Bonder and J. D. Rossi, Blow-up vs. spurious steady solutions, *Proc. Amer. Math. Soc.*, **129(1)** (2001), 139-144.
[23] J. Fernandez Bonder P. Groisman and J. D. Rossi, On numerical blow-up sets, *Proc. Amer. Math. Soc.*, **130(7)** (2002), 2049-2055.
[24] R. Ferreira, P. Groisman and J. D. Rossi, Numerical blow-up for the porous medium equation with a source, *Numer. Methods PDE.*, **20(4)** (2004), 552-575.
[25] V. Galaktionov and J. L. Vazquez, The problem of blow-up in nonlinear parabolic equation. *Current developments in PDE (Temuco,1999), Discrete contin. Dyn. Syst.*, **A 8(2)** (2002), 399-433.
[26] P. Groisman, Totally discrete explicit and semi-implicit Euler methods for a blow-up problem in several space dimension, *Computing*, **76(3-4)** (2006), 325-352.
[27] P. Groisman and J. D. Rossi, Dependance of the blow-up time with respect to parameters and numerical approximations for a parabolic problem, *Asympt. Anal.*, **37** (2004), 79-91.
[28] P. Groisman and J. D. Rossi, Asymptotic behaviour for a numerical approximation of a parabolic problem with blowing up solutions, *Comput. Appl. Math.*, **135(1)** (2001), 135-155.
[29] O. A. Ladyzenskaya, V. A. Solonnikov and N. N. Ural' Ceva, Linear and quasilinear equations of parabolic type, *Trans. Math. Monogr.*, **23** AMS, Providence, RI, (1968).
[30] M. N. Le Roux, Semidiscretization in time of nonlinear parabolic equations with blow-up solution, *SIAM J. Numer. Anal.*, **131(1)** (1994), 170-195.
[31] M. N. Le Roux, Semidiscretization in time of a fast diffusion equations with blow-up solution, *J. Math. Anal. Appl.*, **137(2)** (1989), 354-370, DOI:https://doi.org/10.1016/0022-247X(89)90251-5.
[32] I. Mai and K. Mochizuki, On blow-up of solutions for quasilinear degeneate parabolic equations, *Publ. RIMS. Kyoto Univ.*, **27(5)** (1991), 695-709.
[33] T. Nakagawa, Blowing up on the finite difference solution to $u_t = u_{xx} + u^2$, *Applied Mathematics and Optimization* ., **2(4)** (1975), 337-350.
[34] T. K. Boni and Halima. Nachid, Blow-Up For Semidiscretizations Of Some Semilinear Parabolic Equations With Nonlinear Boundary Conditions, *Rev. Ivoir. Sci. Tech,***11** (2008), 61-70.
[35] T. K. Boni, Halima. Nachid and Nabongo Diabate, Blow-Up For Discretization Of A Localized Semilinear Heat Equation, *Analele Stiintifice Ale Univertatii,* **2** (2010).
[36] Halima. Nachid, Quenching For Semi Discretizations Of A Semilinear Heat Equation With Potentiel And General Non Linearities. *Revue D'analyse Numerique Et De Theorie De L'approximation,* **2** (2011), 164- 181.
[37] Halima. Nachid, Full Discretizations Of Solution For A Semilinear Heat Equation With Neumann Boundary Condition. *Research and Communications in Mathematics and Mathematical Sciences,***1** (2012), 53-85.
[38] Halima. Nachid, Behavior Of The Numerical Quenching Time With A Potential And General Nonlinearities. *Journal of Mathematical Sciences Advances and Application,***15** (2012), 81-105.
[39] M. H. Protter and H. F. Weinberger, Maximum principles in differential equations, *Prentice Hall, Englewood Cliffs, NJ*, (1967).

[40] P. Quittner and P. Souplet, Superlinear parabolic problems, Blow-up, Global existence and Steady States Series: *Birkhuser Advanced Tests/ Basler Lehrbcher*, (2007).
[41] A. M. Stuart and M. S. Floater, On the computation of blow-up, *Euro. J. Appl. Math.*, **1(1)** (1990), 47-71.
[42] A. Samarski, V. A Galaktionov, S. P. Kurdyunov and A. P. Milailov, Blow-up in quasilinear parabolic equations *Walter de Gruyter, Berlin*, (1995).
[43] R. Suzuki, On blow-up sets and asymptotic behavior of interface of one dimensional quasilinear degenerate parabolic equation, *Publ. RIMS. Kyoto Univ.*, **27** (1991), 375-398.
[44] P. Souplet, Uniform Blow-up profiles and boundary behavior for diffusions equations with nonlocal nonlinear source, *J. Diff. Equat.*, **153(2)** (1999), 374-406.
[45] P. Souplet, Blow-up in nonlocal reaction-diffusion equations, *SIAM J. Math. Anal.*, **29(6)** (1998), 1301-1334.
[46] L. Wang and Q. Chen, The asymptotic behavior of blow-up solution of localized nonlinear equation, *J. Math. Anal. Appl.*, **200(2)** (1996), 315-321.
[47] W. Walter, Differential-und Integral-Ungleichungen, *Springer, Berlin*, (1964).
[48] F. B. Weissler, An L^{∞} blow-up estimate for a nonlienar heat equation, *Communications on Pure and Applied Mathematics.*, **38(3)** (1985), 291-295.

Received 22 January 2021

[1] Université Alassane Ouattara de Bouaké, UFR-SED, 01 BP V 18 Bouaké 01, (Côte d'Ivoire)
[2] Université Nangui Abrogoua, UFR-SFA, Département de Mathématiques et Informatiques, 02 BP 801 Abidjan 02, (Côte d'Ivoire), International University of Grand-Bassam Route de Bonoua Grand-Bassam BP 564 Grand-Bassam , (Côte d'Ivoire) et Laboratoire de Modélisation Mathématique et de Calcul Économique LM2CE settat, (Maroc).
[3] Institut National Polytechnique Houphouët-Boigny de Yamoussoukro, BP 2444 Yamoussoukro, (Côte d'Ivoire)
Email address: [1] nkrasoft@yahoo.fr, [2] halimanachid@yahoo.fr, nachid.h@iugb.edu.ci, [3] latoureci@gmail.com