

STUDY OF AN OPTIMAL CONTROL OF A FRICTIONLESS CONTACT PROBLEM

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ABSTRACT. We consider a mathematical model which describes a contact between a nonlinear elastic body and a deformable foundation. The contact is modelled with a normal compliance condition. The goal of this paper is to study an optimal control problem which consists of leading the stress tensor as close as possible to a given target, by acting with a control on the boundary of the body. We proceed with an optimal control problem for which we prove the existence of a solution. Finally, we consider a regularized control problem and prove a convergence result.

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