

**DIFFERENTIAL SANDWICH THEOREMS FOR MULTIVALENT  
ANALYTIC FUNCTIONS DEFINED BY CONVOLUTION STRUCTURE  
WITH GENERALIZED HYPERGEOMETRIC FUNCTION**

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**ABSTRACT.** In this present paper, we obtain some applications of first order differential subordination and superordination results involving Hadamard product for multivalent analytic functions with generalized hypergeometric function in the open unit disk. These results are applied to obtain sandwich results. Many of the well-known and new results are shown to follow as special cases of our results.

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