

BOUNDARY OPTIMAL CONTROL OF A FRICTIONLESS CONTACT PROBLEM WITH LOCKING MATERIALS

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ABSTRACT. In this paper, we consider a frictionless unilateral contact between a nonlinear elastic locking body and a rigid foundation. The goal is to study an optimal control problem, which consists to approach a desired displacement by acting with a control on the boundary of the body. We present an optimal control problem which admits at least one solution. We also introduce the penalized control problem, for which we study the convergence when the penalization parameter tends to zero.

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