

## COMMON FIXED POINT THEOREMS IN PARTIAL $b$ -METRIC SPACES INVOLVING SIMULATION FUNCTIONS

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ABSTRACT. In this work, we prove in a partial  $b$ -metric space a common fixed point theorem for two pairs of mappings that satisfy a generalized contraction condition which involves a simulation function. Our result generalizes and extends several results from the existing literature.

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