

OPTIMAL CONTROL OF A FRICTIONLESS CONTACT PROBLEM IN ISOTROPIC INCOMPRESSIBLE LINEAR ELASTICITY

TOUZALINE AREZKI

ABSTRACT. We consider a mathematical model which describes a static contact between an incompressible linearly elastic isotropic body and a rigid foundation. We assume that the contact is frictionless with Signorini's conditions. We derive a variational formulation and prove its unique weak solvability. We state an optimal control problem which consists of leading the displacement field as close as possible to a desired displacement by acting with a control on the boundary of the body. Then, we study a penalized control problem and prove a convergence result.

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FACULTY OF MATHEMATICS, USTHB, LABORATORY OF DYNAMICAL SYSTEMS, BP 32 EL ALIA, BAB-EZZOUAR, 16111, ALGIERS, ALGERIA.
 Email address: ttouzaline@yahoo.fr