APPROXIMATION BY COMPLEX ACTIVATED SINGULAR INTEGRAL OPERATORS

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ABSTRACT. In this work we study the activated complex-valued singular integral operators over the real line regarding their approximation to the unit operator with rates in the L_p norm, $1 \le p \le \infty$. The related established inequalities involve the higher order L_p modulus of smoothness of the engaged function or its higher order derivative. Also we study the complex-valued fractional general singular integral operators on the real line, regarding their approximation to the unit operator with rates in the uniform norm. The related established inequalities involve the higher order moduli of smoothness of the associated right and left Caputo fractional derivatives of the engaged function. The related simultaneous approximations are also studied in full detail.

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