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NEW SUBCLASS OF HARMONIC UNIVALENT FUNCTIONS DEFINED BY $Q-$CALCULUS OPERATORS

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Abstract. The $q-$difference calculus or quantum calculus is the $q-$extension of the ordinary fractional calculus and it was initiated at the beginning of the 19th century. The theory of $q-$calculus operators is used in various areas of science such as ordinary fractional calculus, optimal control and in the geometric function theory of complex analysis and was initially developed by Jackson [7, 8]. The purpose of the present paper is to apply $q-$calculus operators to a new subclass of harmonic univalent functions. We establish some interesting coefficient conditions, distortion theorems, covering results, extreme points, convolution conditions and convex combinations for $f \in \overline{\mathfrak{L}_H}(\beta; q, m, n)$. Finally, we apply the $q-$analogous to Bernardi's integral operator.

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