Analele Universităţii Oradea
Fasc. Matematica, Tom XXX (2023), Issue No. 1, 129–147

PARAMETRIZED ERROR FUNCTION BASED BANACH SPACE VALUED MULTIVARIATE MULTI LAYER NEURAL NETWORK APPROXIMATIONS

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Abstract. Here we describe multivariate quantitative approximations of Banach space valued continuous multivariate functions on a box or $\mathbb{R}^N$, $N \in \mathbb{N}$, by the multivariate normalized, quasi-interpolation, Kantorovich type and quadrature type neural network operators. We treat also the case of approximation by iterated operators of the last four types, these correspond to hidden multi-layer neural networks. The approximations are derived by establishing multidimensional Jackson type inequalities involving the multivariate modulus of continuity of the engaged function or its high order partial derivatives. Our multivariate operators are defined by using a multidimensional density function induced by a parametrized error function. The approximations are pointwise and uniform. The related feed-forward neural network starts with one hidden layer.

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2010 *Mathematics Subject Classification.* 41A17, 41A25, 41A30, 41A36.

Key words and phrases. Parametrized error function, multivariate neural network approximation, quasi-interpolation operator, Kantorovich type operator, quadrature type operator, multivariate modulus of continuity, abstract approximation, iterated multi layer neural network approximation.

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Received 1 February 2023

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