HYPERBOLIC TANGENT LIKE INDUCED BANACH SPACE VALUED NEURAL NETWORK APPROXIMATION

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ABSTRACT. Here we examine the univariate quantitative approximation of Banach space valued continuous functions on a compact interval or all the real line by quasi-interpolation Banach space valued neural network operators. We perform also the related Banach space valued fractional approximation. These approximations are derived by establishing Jackson type inequalities involving the modulus of continuity of the engaged function or its Banach space valued high order derivative or fractional derivaties. Our operators are defined by using a density function induced by a general Hyperbolic Tangent Like sigmoid function. The approximations are pointwise and with respect to the uniform norm. The related Banach space valued feed-forward neural networks are with one hidden layer.

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