

BEHAVIOR OF THE BLOW-UP TIME OF A NONLINEAR HYPERBOLIC EQUATION WITH DIRICHLET BOUNDARY CONDITIONS AND A POTENTIAL

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ABSTRACT. The aim of this paper is the study of the solution of a nonlinear hyperbolic equation subject to Dirichlet boundary conditions and a potential. Under suitable hypotheses, we show that the solution of the above problem blows up in a finite time, and its blow-up time tends to the one of the solution of a differential equation when a certain parameter goes to zero. We give some numerical results to illustrate our analysis at the end of our study.

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