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ON SQUARE-MEAN S-ASYMPTOTICALLY BLOCH TYPE PERIODICITY OF SOME STOCHASTIC EVOLUTION EQUATIONS

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Abstract. This paper introduces the concept of square-mean pseudo S-asymptotically Bloch type periodic for stochastic processes and establishes some composition and convolution theorems for such stochastic processes. In addition, it presents the notion and properties of square-mean weighted pseudo S-asymptotically Bloch-type periodic functions similarly. Finally, we investigate the existence and uniqueness of square-mean pseudo S-asymptotically ω-periodic mild solutions of some stochastic integrodifferential equations driven by fractional Brownian motion.

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