

APPLICATION OF GEGENBAUER POLYNOMIALS TO BI-UNIVALENCY INVOLVING THE SIGMOID ACTIVATION FUNCTION

A.A. AYENI¹, I.T. AWOLERE², A.T. OLADIPO³ AND S.R. SWAMY⁴

ABSTRACT. The Gegenbauer polynomials with two variables, and the sigmoid function whose operation is similar to that of biological central nervous system are investigated. The authors in this work applied the earlier mentioned two functions using subordination principle to derive the early coefficients, Fekete-Szego inequality, second Hankel and Toeplitz determinants of the defined function classes. Several consequences of our results are pointed out.

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¹ FEDERAL COLLEGE OF EDUCATION (SPECIAL), DEPARTMENT OF MATHEMATICS, P.M.B. 1089, OYO, OYO STATE NIGERIA.

² OLUSEGUN AGAGU UNIVERSITY OF SCIENCE AND TECHNOLOGY, OKITI PUPA, DEPARTMENT OF MATHEMATICAL SCIENCE, OKITI PUPA, ONDO STATE, NIGERIA.

³ LADOKE AKINTOLA UNIVERSITY OF TECHNOLOGY, OGBOMOSO, DEPARTMENT OF PURE AND APPLIED MATHEMATICS, P.M.B. 4000, OGBOMOSO, OYO STATE, NIGERIA.

⁴ DEPARTMENT OF INFORMATION SCIENCE AND TECHNOLOGY, ACHARYA INSTITUTE OF TECHNOLOGY, BENGALURU - 560 107, KARNATAKA, INDIA.

Email address: ¹ ayen.adeniyi1501@fcesoyo.edu.ng, ² awolereibrahim01@gmail.com, ³ atoladipo@lautech.edu.ng, ⁴ swamy2704@acharya.ac.in