

AN EXTENSION OF THE OPEN DOOR LEMMA

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ABSTRACT. We research the real part of holomorphic functions based on Miller and Mocanu's remarkable result which is now known as the Open Door Lemma. It provides a sufficient condition for an analytic function on the disk belonging to Carathéodory class. In this paper, we give a sufficient condition such that the real part of holomorphic functions in the disk to be greater than α ($\alpha > 0$). The result presented here would extend previously results, obtained by Li and Sugawa. Moreover, we also discuss the image of the open door function obtained in the main result.

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2010 *Mathematics Subject Classification*. Primary 30C45, 30C50, 30C80.

Key words and phrases. Open Door Lemma, analytic function, subordination.

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Received 15 February 2022

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