

CONTINUITY OF THE NUMERICAL QUENCHING TIME IN A SEMILINEAR HEAT EQUATION

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ABSTRACT. This paper concerns the study of the numerical approximation for the following initial-boundary value problem:

$$\begin{aligned}u_t &= u_{xx} - u^{-p}, & x \in (0, 1), & \quad t \in (0, T_q), \\u_x(0, t) &= 0, \quad u_x(1, t) = 0, & t \in (0, T_q), \\u(x, 0) &= u_0(x) > 0, & x \in [0, 1].\end{aligned}$$

Under some assumptions, we prove that the solution of a semidiscrete form of the above problem quenches in a finite time and estimate its numerical quenching time. We also prove the continuity of the quenching time as a function of the initial datum. Finally, we give some numerical experiments to illustrate our analysis.

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