

**EXISTENCE RESULTS FOR FRACTIONAL ORDER FUNCTIONAL
DIFFERENTIAL EQUATIONS WITH INFINITE DELAY IN THE SENSE
OF THE DEFORMABLE DERIVATIVE**

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ABSTRACT. In this article, we discuss the existence and uniqueness of solutions for initial value problems of fractional order functional and neutral functional differential equations with infinite delay. We use the deformable derivative introduced in 2017 by Zulfeqarr et. al (see [21]). Our results are obtained using the Banach fixed point theorem and the nonlinear alternative Leray–Schauder type theorem. We provide an example as an illustration of the main results.

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