ASYMPTOTIC ANALYSIS FOR A REACTION DIFFUSION EQUATION WITH A LOCALIZED NONLINEAR SOURCE TERM AND HOMOGENEOUS BOUNDARY CONDITIONS

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Abstract. In this paper, we study a numerical approximation for the following localized nonlinear reaction-diffusion equation
\[
\begin{cases}
  u_t(x, t) = u_{xx}(x, t) + f(u(a_0, t)), & (x, t) \in (-1, 1) \times (0, T), \\
  u(-1, t) = 0, & u_x(1, t) = 0, & t \in (0, T), \\
  u(x, 0) = u_0(x) \geq 0, & x \in [-1, 1],
\end{cases}
\]
where \( f(s) \) is a positive, increasing, \( c^2 \) convex function for the nonnegative values of \( s \), \( \int_{\mathbb{R}^+} f(s) ds < +\infty \) for \( c > 0 \), \( a_0 = 1 \), \( u_0 \in C^2([-1, 1]) \), \( u_0(-1) = 0 \), \( u_0'(1) = 0 \).

We find some conditions under which the solution of a discrete form of the above problem blows up in a finite time and estimate its discrete blow-up time. We also prove the convergence of the numerical blow-up time to the theoretical one. Finally, we give some numerical results to illustrate our analysis.

References


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