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BOUNDEDNESS AND STABILITY IN THIRD ORDER NONLINEAR DIFFERENTIAL EQUATIONS WITH BOUNDED DELAY

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Abstract. By defining an appropriate complete Lyapunov functional, sufficient conditions are established which guarantee uniform asymptotic stability of the trivial solution and uniform ultimate boundedness of solutions of equation (1.1). The results obtained in this investigation extend many existing and exciting results on nonlinear third order delay differential equations.

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