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# VARIATIONAL ANALYSIS OF A UNILATERAL CONTACT PROBLEM WITH ADHESION AND FRICTION

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Abstract. The aim of this paper is to study a quasistatic contact between an elastic body and an obstacle. The constitutive law is nonlinear and the contact is modelled with Signorini's conditions with a gap and adhesion, associated with a version of Coulomb's law of dry friction. The adhesion between contact surfaces is described by a first-order differential equation. We establish a variational formulation of the mechanical problem and prove an existence and uniqueness result. The technique of the proof is based on arguments of time-dependent variational inequalities, differential equations and Banach fixed-point theorem.

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