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NUMERICAL BLOW-UP FOR SOLUTIONS OF SEMILINEAR HEAT EQUATIONS WITH SMALL DIFFUSION

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Abstract. This paper concerns the study of the numerical approximation for the following initial-boundary value problem:

$$\begin{cases} u_t = \varepsilon u_{xx} + e^u, \quad x \in (0,1), \quad t \in (0,T), \\ u(0,t) = 0, \quad u(1,t) = 0, \quad t \in (0,T), \\ u(x,0) = 0, \quad x \in [0,1], \end{cases}$$

where $\varepsilon > 0$. We prove that for small ε the solution of a semidiscrete form of above problem blows up in a finite time and the semidiscrete blow-up time goes to one as the diffusion parameter ε tends to zero. We also show that the semidiscrete blow-up time in certain cases converges to the real one when the mesh size tends to zero. Finally, we give some numerical experiments to illustrate our analysis.

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