

EXISTENCE AND EXPONENTIAL DECAY OF SOLUTIONS FOR THE VARIABLE-COEFFICIENT WAVE EQUATIONS

YAMNA BOUKHATEM¹ AND BENYATTOU BENABDERRAHMANE²

ABSTRACT. In this paper, we consider a variable-coefficient wave equation with damping and source terms. Under suitable conditions on the initial data we show that the local solution exists globally in time and we derive the exponential decay of the energy solution without any relation between the elements of the matrix A and the damping and source terms.

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¹ LABORATORY OF COMPUTER SCIENCES AND MATHEMATICS, FACULTY OF SCIENCES, UNIVERSITY OF LAGHOUAT, P.O. BOX 37G, LAGHOUAT (03000), ALGERIA, ² FACULTY OF MATHEMATICS AND COMPUTER SCIENCES, MOHAMED BOUDIAF UNIVERSITY - M'SILA (28000), ALGERIA

E-mail address: ¹ byamna@yahoo.fr, ² bbentattou@yahoo.com